RAISEPARTNER
QUANTITATIVE PORTFOLIO MANAGEMENT

The Quant Corner
October 2008

Risk Filtering

Covariance matrix filtering in a decision making framework

WHY DO WE NEED TO FILTER COVARIANCE MATRICES?

A big misunderstanding originates from the gap between two domains of the quantitative finance: the **descriptive statistics** (to explain the risk) and the **decision mathematics** (to allocate the risk). The same words are sometimes used in both contexts, such as the important notion of **robustness**, but do not mean the same thing. As a consequence, risk models which are acknowledged as robust in a descriptive risk analysis framework will not be suitable in a decision making context, and vice versa.

For example, **empirical estimations** might be the best estimates of the covariance matrices in a statistical approach, but using them in a portfolio optimization context can be highly dangerous. Indeed, statistical analysis shows that empirical covariance matrix estimators cannot capture properly the small contributions to risk, especially when using noisy market data.

These small contributions to risk can be neglected when **explaining** the risk of a portfolio. Indeed, 95% or so of the information is sufficient to analyse/decompose the risk. Furthermore, such risk management approaches use large estimation windows (1 year or more) to compute the covariance matrix, which allows to reduce statistical errors.

But this is not the case in a **decision making framework** such as portfolio optimization. Indeed, to design a reactive (optimization-oriented) risk model and take the right allocation decision, we need to use shorter estimation windows, especially in times of highly volatile markets. The Random Matrix Theory tells us that the lesser data we use, the more underestimated the small contributions to risk.

When optimizing a portfolio, underestimating these contributions (i.e. the smallest eigenvalues) can be very dangerous. Indeed, it implies that there exists a theoretical portfolio with a volatility close to zero[1]! If this portfolio happens to have a positive expected return, then its Sharpe ratio will be $+\infty$, and the optimizer will be lured by this aberrant portfolio. In the end, this means that **the portfolio optimization process is essentially driven by noise**… hence implying very large turnovers and operational risk at each reallocation.

WHEN DO WE NEED TO FILTER COVARIANCE MATRICES?

Matrix filtering is especially needed in situations where **the specific structural properties of covariance matrices are destroyed**:

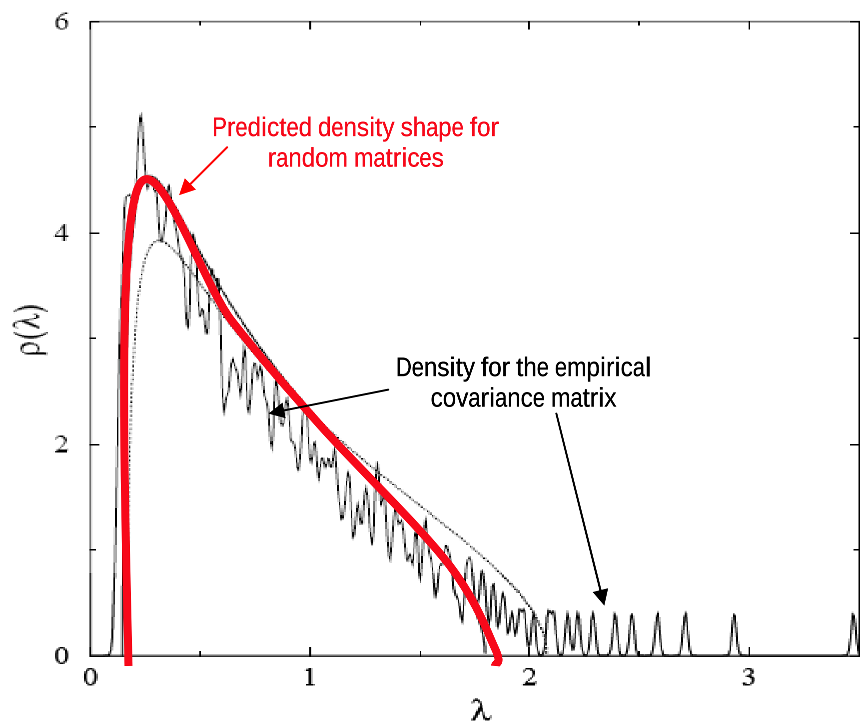
[1] The eigenvalue represents the variance of the portfolio defined by the corresponding normalized eigenvector. Hence, an eigenvalue close to zero implies that this portfolio has a volatility close to zero.

- Heterogeneous data series (asynchronicity due to large and diversified investment universes, multi data-provider…)
- Aggregation of local information into a global risk model (estimation of local matrices by independent statistical schemes)
- Stress testing risk matrices (in a risk management framework)
- Prior views or intervals constraints on your matrix (volatilities, subset of the whole universe)

SOME RANDOM MATRIX THEORY BACKGROUND

Random Matrix Theory predicts a specific shape for the density of the eigenvalues λ of a 100% random matrix.

In the example below (from **[3]**), we compare the density of a sample covariance matrix (the covariance matrix of the returns of the S&P500 index components) with the density of a random matrix. The chart hereafter shows the predicted density shape for random matrices (in red) vs. the density of an empirical covariance matrix (in black):

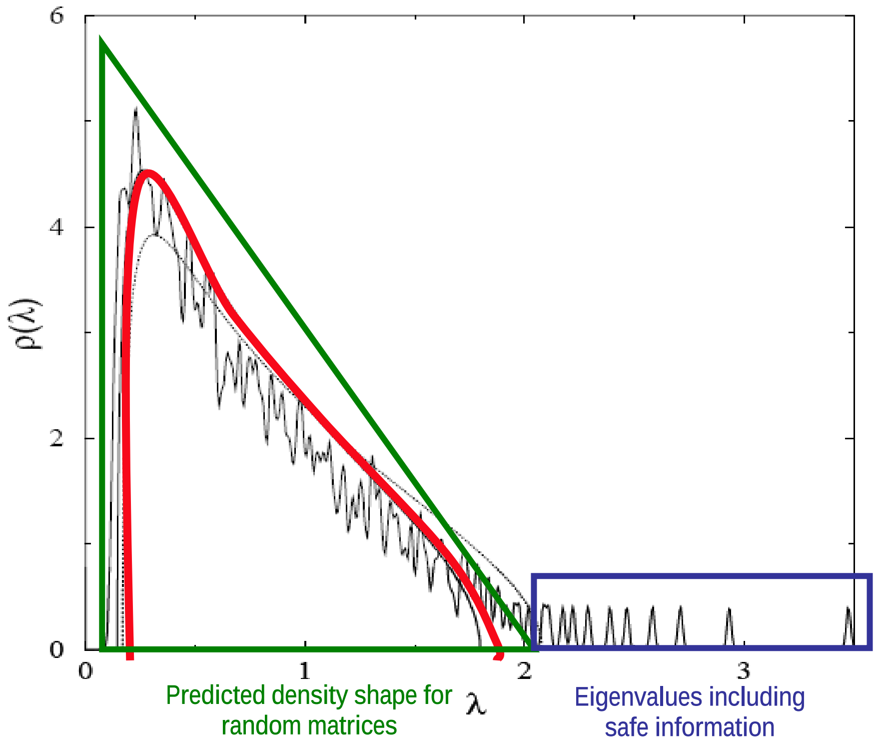


Compared eigenvalues densities (random matrix vs. sample matrix), from [3]

This chart suggests that the eigenvalues of the sample matrix that are smaller than a certain threshold (around 2 in this example) might include much noise.

NB: we do not claim that this part of the density contains nothing but noise, that would be a shortcut. Indeed, every 100% random matrix will have a density that fits this shape, but this does not mean that this is reciprocal: this chart cannot ensure that most of the information is noise. But in a conservative approach, we will consider that it might be.

Based on this hypothesis, **only the upper part of the spectrum embeds "safe" market information**, as highlighted below:



Noise vs. safe market information

This shows that using covariance matrices in a portfolio optimization framework **requires a matrix correction process to filter the noise** and distinguish safe market information.

EXISTING PRACTICES

A straightforward solution would be to simply increase **the smallest eigenvalues** to a specified threshold. This method would restore the semi-definite positivity property of the covariance matrix, but it does not allow to control explicitly both the deformation applied to the matrix and its conditioning.

Ledoit & Wolf propose in **[2]** a **shrinkage method** consisting in finding the right trade-off between bias and estimation error. The shrinked matrix is a linear interpolation of the sample covariance matrix and a structured estimate (biased) of the covariance matrix. This method improves the conditioning of the covariance matrix only if the structured estimate is already well conditioned. But it does not allow to control the deformation applied to the sample covariance matrix with explicit constraints.

A CONSERVATIVE-YET-REACTIVE APPROACH

The matrix filtering method provided by RaisePartner allows to build conservative and reactive risk model:

- **reactive**: capturing current trends by reducing the estimation windows and incorporating implied information
- **conservative**: producing robust portfolio decisions by filtering the noise due to the lack/uncertainty of data

We can compare this approach to the robust control model for a fighter aircraft: the model needs to react quickly but cautiously using very short-term information through a very conservative filtering scheme.

Our filtering method consists in **taking the "best candidate" from the descriptive statistics world and making it "cross" towards the decision making world**. In other words, we choose the PSD (Positive Semi-Definite) matrix which is the closest from the best candidate (the empirical covariance matrix for instance) and which verifies the set of required linear constraints.

This method allows to:

- **control the conditioning** of the covariance matrix

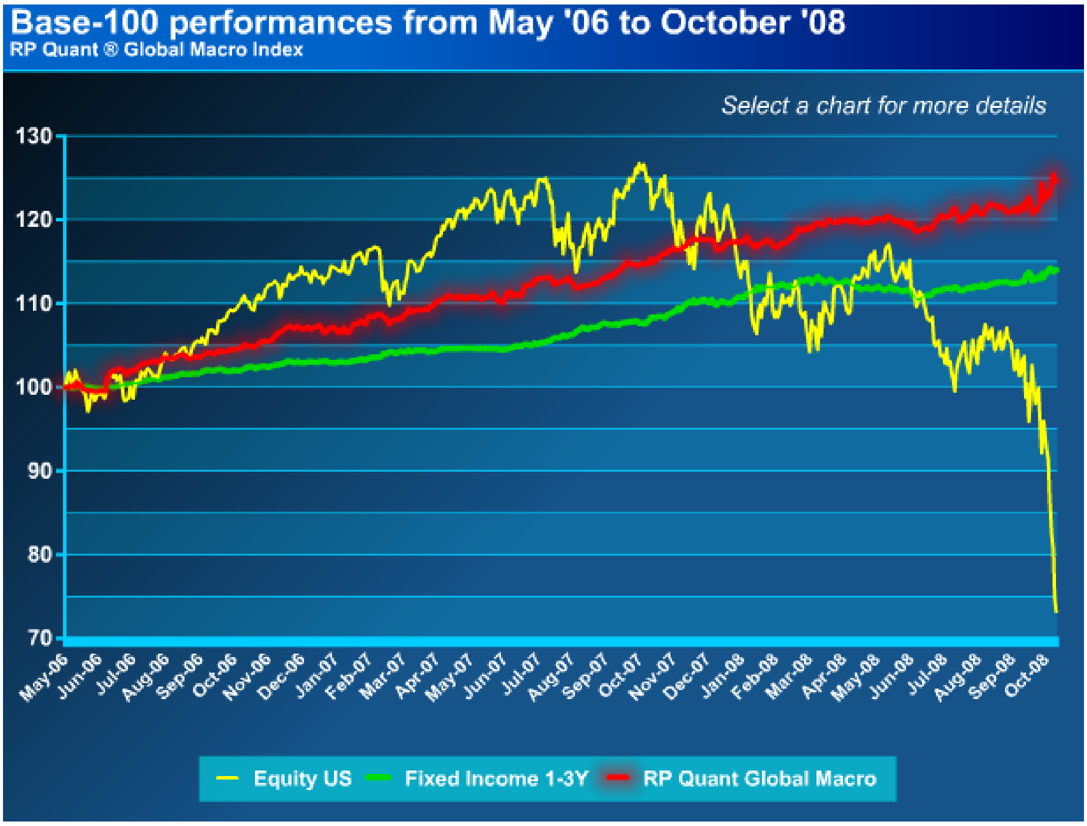
- **control the deformation** of the matrix with a set of constraints such as:
    - preserving volatilities
    - preserving the trace
    - preserving some part of the covariance matrix
    - giving prior views or confidence intervals on some part of the matrix
    - Including implied information

A REAL-LIFE EXAMPLE: RP QUANT GLOBAL MACRO INDEX

Since its inception in 2006, the RP Quant Global Macro index has been managed with RaisePartner's solutions, embedding this filtering approach described above. It is a theoretical index composed of diversified ETFs.

The RP Quant Global Macro Index is **rebalanced on a weekly basis**, hence the reactivity of the risk model is crucial. The **volatility budget is around 5%,** which requires some robustness in the risk modelling and portfolio optimization approach.

The chart below shows the base-100 performances of the index since May 06. To learn more about the RP Quant Global Macro Index, visit the RP Quant Indices section of our website.



Base-100 performances of the RP Quant Global Macro Index

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[1] Papa Momar Ndiaye, François Oustry and Véronique Piolle. *Semidefinite optimisation for global risk modelling*. Journal of Asset Management 7, 142–153, 2006.

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[3] L. Laloux, P. Cizeau, J-P. Bouchaud and M. Potters. *Noise Dressing of Financial Correlation Matrices*. Phys. Rev. Lett. 83, 1467 - 1470 , 1999.

[4] L. El Ghaoui, F. Oustry and H. Lebret. *Robust Solutions to Uncertain Semidefinite Programs.* SIAM J. Optimization, volume 9, no. 1, 1998.